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Type-2 Fuzzy Non-uniform Rational B-spline Model with Type-2 Fuzzy Data

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ABSTRACT

This paper introduced type-2 fuzzy non-uniform rational B-spline curve (F2NURBS) interpolation model for type-2 fuzzy data points. The curve model is defined using type-2 fuzzy control points, crisp knot and weight. Later, the data points of type-2 fuzzy NURBS curve after type-reduction and defuzzification is discussed. Finally, a numerical example and an algorithm to generate the model are shown at the end of this paper.

Keywords: Type-2 fuzzy non-uniform rational B-spline, interpolation model, type-2 fuzzy control points, type-reduction, defuzzification.

1. Introduction

Real-life situation is full of vagueness and uncertainties. Neglecting those uncertainties would mean fail to describe the true situation of data points. With the method of fuzzy set theory, uncertainties can be transformed into a form of set, which is described by some value of truth. The elements in the domain of uncertainty will match with the value of membership in the close interval 0 and 1. By using this approach, the data will be in fuzzy data form. If uncertainty in uncertainty is considered, then the data will be in the form of type-2 fuzzy data set. For interpolation model, the data points are acquired from the object, and the NURBS interpolation could be used to reproduce the shape of the object such as curve or surface. In this paper, type-2 fuzzy set theory and geometric modeling with uncertainties characteristic is investigated and visualized.

Zadeh (1965) introduced fuzzy set theory to describe uncertainties by using mathematical representation. Later, the concept of linguistic variable is introduced in Zadeh (1975) where linguistic variable can be used to measure words or sentences in language within values 0 and 1. Then Dubois et al. (2000) properly defined the word fuzzy. They also provided a systematic framework to handle uncertainties that occur in human thought. Moreover, the concept of fuzzy numbers was introduced by Dubois and Prade (1980). The operation on sets and fuzzy numbers such as addition, multiplication, intersection and union is referred to Klir and Yuan (1995). Finally, type-2 fuzzy set theory is defined in details by Mendel (2001) where the uncertainty in its membership function is defined as secondary membership grade.

There are some past research that combines fuzzy set theory and geometric modeling such as Jacas et al. (1997) that used fuzzy logic in CAGD for curve and surface design and Anile et al. (2000) discussed about fuzzy B-splines. In addition, researches on fuzzy Bézier curve have been done by Wahab et al. and Zakaria et al. (2001) and have been applied on off-line handwritten signature verification and fuzzy grid data. Fuzzy interpolation of rational cubic Bézier are proposed with different degree of signature blurring in Zakaria and Wahab (2012). Fuzzy interpolation of B-splines curve is then introduced in Karim et al. (2013) and Wahab and Zakaria (2012). Particularly on NURBS, Wahab and Husain (2011a) and Wahab and Husain (2011b) shown fuzzy NURBS curve and surface with fuzzy control points, fuzzy knot and fuzzy weight.

Data points, knots and weights are three important elements to generate NURBS curve or surface. However, when uncertainties occur in either data points, knots or weights, crisp NURBS model fails to express the problem.

Recently, research has been done on type-1 fuzzy Bézier, type-1 fuzzy B-spline, type-1 fuzzy NURBS model as in Anile et al. (2000), Wahab et al., Karim et al. (2013) and Zakaria and Wahab (2013). Nevertheless, investigation on one of the common interpolation method, NURBS curve interpolation that can handle uncertainties in either data points, knots or weights have not been done yet. As a matter of fact, when researchers are dealing with type-2 uncertainty data problem, the existing type-1 fuzzy model is unable to define the type-2 uncertainty (Zakaria et al. (2013a)). Thus, research on type-2 fuzzy NURBS curve interpolation is necessary to be investigated.

2. Type-2 Fuzzy NURBS Curve Modeling

The proposed type-2 fuzzy NURBS curve model provides better curve interpolation that can handle complex uncertainties and enables it to reproduce a more desired and smoother curve. Particularly, in curve interpolation, the developed model can handle type-2 uncertainty data through type-2 fuzzy data points in curve interpolation. The propose model allows the users to tuned the interpolation curve by adjusting knot and weight to obtain a perfect desired curve. As a result, the proposed method is an important contribution to CAGD especially in reverse engineering. The development of type-2 fuzzy NURBS based on type-2 fuzzy control points was discussed in Zakaria et al. (2013a) and Zakaria et al. (2013b). Undoubtedly, the advantage of type-2 fuzzy in geometric modeling is it could define uncertainties in uncertainties. Thus, type-2 fuzzy set is an appropriate method to describe uncertainties in data points.

3. Type-2 Fuzzy Points

Definition 3.1. *Let* $P = \{x|x \text{ type-2 fuzzy point}\}$ *and* ${}^{T2}\overset{\leftrightarrow}{P} = \{P_i|P_i \text{ point}\}$ *is a set of type-2 fuzzy point with* $P_i \in P \subset X$*, where* X *is universal set. The membership function* $\mu_P(P_i) : P \to [0,1]$ *is defined as* $\mu_P(P_i) = 1$ *and formulated by* ${}^{T2}\overset{\leftrightarrow}{P} = \{(P_i, \mu_P(P_i)|P_i \in \mathbb{R}\}$*. Thus,*

$$\mu_P(P_i) = \begin{cases} 0 & \text{if } P_i \notin X, \\ c \in (0,1) & \text{if } P_i \overset{\leftrightarrow}{\in} X, \\ 1 & \text{if } P_i \in X. \end{cases} \tag{1}$$

where $\mu_P(P_i) = \langle \mu_P({}^{T2L}\overset{\leftrightarrow}{P}_i), \mu_P(P_i), \mu_P({}^{T2R}\overset{\leftrightarrow}{P}_i)\rangle$ *with* $\mu_P({}^{T2L}\overset{\leftrightarrow}{P}_i)$ *are left;* $\mu_P({}^{T2R}\overset{\leftrightarrow}{P}_i)$

are right footprint of membership values with $\mu_P({}^{T2L}\overset{\leftrightarrow}{P_i}) = \langle \mu_P({}^{L}\overset{\leftrightarrow}{P_i^{\leftarrow}}),\, \mu_P({}^{L}\overset{\leftrightarrow}{P_i^{\rightarrow}})\rangle$ *and* $\mu_P({}^{L}\overset{\leftrightarrow}{P_i^{\leftarrow}})$ *are lower bound of the left footprint;* $\mu_P({}^{L}\overset{\leftrightarrow}{P_i^{\rightarrow}})$ *is the upper bound of left footprint.* $\mu_P({}^{R}\overset{\leftrightarrow}{P_i^{\leftarrow}})$ *are membership grade of lower bound of right footprint;* $\mu_P({}^{R}\overset{\leftrightarrow}{P_i^{\rightarrow}})$ *is the membership grade of upper bound of right footprint.*

Type-2 fuzzy point can also be written as ${}^{T2}\overset{\leftrightarrow}{P} = \{{}^{T2}\overset{\leftrightarrow}{P_i} : i = 0, 1, 2, \ldots, n\}$. For all i, ${}^{T2}\overset{\leftrightarrow}{P_i} = \langle {}^{T2L}\overset{\leftrightarrow}{P_i}, P_i, {}^{T2R}\overset{\leftrightarrow}{P_i}\rangle$ with ${}^{T2L}\overset{\leftrightarrow}{P_i} = \langle {}^{L}\overset{\leftrightarrow}{P_i^{\leftarrow}}, {}^{L}\overset{\leftrightarrow}{P_i^{\rightarrow}}\rangle$ where ${}^{L}\overset{\leftrightarrow}{P_i^{\leftarrow}}$ are lower bound of left footprint; ${}^{L}\overset{\leftrightarrow}{P_i^{\rightarrow}}$ are upper bound of left footprint of type-2 fuzzy point and ${}^{T2R}\overset{\leftrightarrow}{P_i} = \langle {}^{R}\overset{\leftrightarrow}{P_i^{\leftarrow}}, {}^{R}\overset{\leftrightarrow}{P_i^{\rightarrow}}\rangle$ where ${}^{R}\overset{\leftrightarrow}{P_i^{\leftarrow}}$, and ${}^{R}\overset{\leftrightarrow}{P_i^{\rightarrow}}$ are lower and upper bound of type-2 fuzzy point respectively. Figure 1 shows a process of obtaining type-2 fuzzy points/knot/weights.

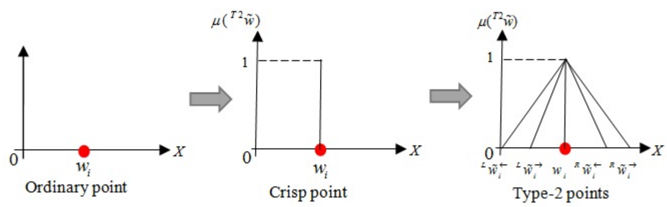

Figure 1: Process of obtaining type-2 fuzzy points/knots/weights

Definition 3.2. *Let* ${}^{T2}\overset{\leftrightarrow}{P}$ *be the set of type-2 fuzzy points and* ${}^{T2}\overset{\leftrightarrow}{P_i} \in {}^{T2}\overset{\leftrightarrow}{P}$ *where* $i = 0, 1, \ldots, n$ *and* $n+1$ *is the number of point.* α*-cut operation on type-2 fuzzy points* ${}^{T2}\overset{\leftrightarrow}{P_i^{\alpha}}$ *is defined as,*

$$\begin{aligned}
{}^{T2}\overset{\leftrightarrow}{P_i^{\alpha}} &= \left\langle \langle {}^{L}\overset{\leftrightarrow}{P_i^{\alpha\leftarrow}}; {}^{L}\overset{\leftrightarrow}{P_i^{\alpha\rightarrow}}\rangle, P_i, \langle {}^{R}\overset{\leftrightarrow}{P_i^{\alpha\leftarrow}}; {}^{R}\overset{\leftrightarrow}{P_i^{\alpha\rightarrow}}\rangle \right\rangle \\
&= \Big\langle \big[(P_i - \langle {}^{L}\overset{\leftrightarrow}{P_i^{\alpha\leftarrow}}; {}^{L}\overset{\leftrightarrow}{P_i^{\alpha\rightarrow}}\rangle)\alpha + \langle {}^{L}\overset{\leftrightarrow}{P_i^{\alpha\leftarrow}}; {}^{L}\overset{\leftrightarrow}{P_i^{\alpha\rightarrow}}\rangle\big], P_i, \qquad (2) \\
&\qquad \big[-(\langle {}^{R}\overset{\leftrightarrow}{P_i^{\alpha\leftarrow}}; {}^{R}\overset{\leftrightarrow}{P_i^{\alpha\rightarrow}}\rangle - P_i)\alpha + \langle {}^{R}\overset{\leftrightarrow}{P_i^{\alpha\leftarrow}}; {}^{R}\overset{\leftrightarrow}{P_i^{\alpha\rightarrow}}\rangle\big]\Big\rangle
\end{aligned}$$

with specific α *value chosen, where* $\alpha \in (0, 1]$. *Type-2 fuzzy points after* $\alpha-$*cut can be calculated by using Equation (2). Moreover, Equation (2) can also be applied when the points are type-1 fuzzy points, where* ${}^{L}\overset{\leftrightarrow}{P_i^{\alpha\leftarrow}} = {}^{L}\overset{\leftrightarrow}{P_i^{\alpha\rightarrow}}$ *and* ${}^{R}\overset{\leftrightarrow}{P_i^{\alpha\leftarrow}} = {}^{R}\overset{\leftrightarrow}{P_i^{\alpha\rightarrow}}$.

Definition 3.3. *Let* ${}^{T2}\overset{\leftrightarrow}{P}_i$ *be a set of* $(n+1)$ *type-2 fuzzy points, type-reduction method of* ${}^{T2}\overset{\leftrightarrow}{P}_i$ *after* $\alpha-$*cut of* ${}^{T2}\overset{\leftrightarrow}{P}_i^{\alpha}$, *is defined as,*

$$\bar{\overset{\leftrightarrow}{P}}{}^{\alpha} = \{\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha} = \langle {}^{L}\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha}, P_i, {}^{R}\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha}\rangle; i = 0, 1, \ldots, n\} \tag{3}$$

where ${}^{L}\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha}$ *is left type-reduction of* $\alpha-$*cut type-2 fuzzy points,* ${}^{L}\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha} = \frac{1}{2}\sum_{i=0}^{n} \langle {}^{L}\overset{\leftrightarrow}{P}{}_i^{\alpha\leftarrow} + {}^{L}\overset{\leftrightarrow}{P}{}_i^{\alpha\rightarrow}\rangle$, P_i *is the crisp point and* ${}^{R}\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha}$ *is right type-reduction of* $\alpha-$*cut of type-2 fuzzy points,* ${}^{R}\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha} = \frac{1}{2}\sum_{i=0}^{n} \langle {}^{R}\overset{\leftrightarrow}{P}{}_i^{\alpha\leftarrow} + {}^{R}\overset{\leftrightarrow}{P}{}_i^{\alpha\rightarrow}\rangle$.

Definition 3.4. $\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha}$ *is the type-reduction method after -cut process is applied for every type-2 fuzzy points, then defuzzification process for* $\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha}$ *is denoted by* $\bar{\bar{P}}{}_i^{\alpha}$. *If every* $\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha} \in \bar{\overset{\leftrightarrow}{P}}{}^{\alpha}$, $\bar{\bar{P}}{}_i^{\alpha} = \{\bar{\bar{P}}{}_i^{\alpha}\}$ *for* $i = 0, 1, \ldots, n$. *So, the defuzzification process can be formulated as,*

$$\bar{\bar{P}}{}_i^{\alpha} = \frac{1}{3}\sum_{i=0}^{n} \langle {}^{L}\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha}, P_i, {}^{R}\bar{\overset{\leftrightarrow}{P}}{}_i^{\alpha}\rangle \tag{4}$$

4. Type-2 Fuzzy NURBS Curve Interpolation

Similar to type-2 fuzzy NURBS curve model, the contradiction of the crisp boundary in type-1 fuzzy model is insufficient to describe fuzziness that occur in the membership grade. When researchers are uncertain of the membership grade of type-1 fuzzy sets, in this case, type-1 fuzzy NURBS curve interpolation, a type-2 fuzzy NURBS curve interpolation with type-2 fuzzy data points is constructed instead of type-1 fuzzy set to encounter type-2 fuzzy problem. The type-2 fuzzy data points are defined by type-2 fuzzy number and type-2 fuzzy relation. Type-2 fuzzy data point is suitable to handle complex uncertainty data as in Zakaria et al. (2013a) and Zakaria et al. (2013b).

Definition 4.1. *Interpolation of NURBS curve with type-2 fuzzy data points are defined as,*

$$T2FNrbI = \{{}^{T2}\overset{\leftrightarrow}{N}rb_{\alpha} | {}^{T2}\overset{\leftrightarrow}{N}rb_{\alpha}, \alpha \in (0, 1]\} \tag{5}$$

where ${}^{T2}\overset{\leftrightarrow}{N}rb_{\alpha} = \big\langle({}^{L}\overset{\leftrightarrow}{N}rb_{\alpha}^{\leftarrow}, {}^{L}\overset{\leftrightarrow}{N}rb_{\alpha}^{\rightarrow}), Nrb_{\alpha}, ({}^{R}\overset{\leftrightarrow}{N}rb_{\alpha}^{\leftarrow}, {}^{R}\overset{\leftrightarrow}{N}rb_{\alpha}^{\rightarrow})\big\rangle$, $({}^{L}\overset{\leftrightarrow}{N}rb_{\alpha}^{\leftarrow}, {}^{L}\overset{\leftrightarrow}{N}rb_{\alpha}^{\rightarrow})$ *and* $({}^{R}\overset{\leftrightarrow}{N}rb_{\alpha}^{\leftarrow}, {}^{R}\overset{\leftrightarrow}{N}rb_{\alpha}^{\rightarrow})$ *represent left and right footprint of type-2 fuzzy NURBS* *interpolation curve after $\alpha-cut$. Nrb_α represents crisp NURBS interpolation curve. $\alpha-cut$ level is defined by user and it is within the half open interval $(0,1]$. By assuming that only one $\alpha-cut$ is applied, five NURBS curve could be generated. Finally, by rewriting the symbol in Equation (5) will yield*

$$^{L}\overset{\leftrightarrow}{N}rbI_{\alpha}^{\leftarrow}(u)=\sum_{i=0}^{n}R_{i,p}(\bar{u}_k)^{L}\overset{\leftrightarrow}{P}_{i}^{\leftarrow}, \tag{6}$$

$$^{L}\overset{\leftrightarrow}{N}rbI_{\alpha}^{\rightarrow}(u)=\sum_{i=0}^{n}R_{i,p}(\bar{u}_k)^{L}\overset{\leftrightarrow}{P}_{i}^{\rightarrow}, \tag{7}$$

$$NrbI_{\alpha}(u)=\sum_{i=0}^{n}R_{i,p}(\bar{u}_k)P_i, \tag{8}$$

$$^{R}\overset{\leftrightarrow}{N}rbI_{\alpha}^{\leftarrow}(u)=\sum_{i=0}^{n}R_{i,p}(\bar{u}_k)^{R}\overset{\leftrightarrow}{P}_{i}^{\leftarrow}, \tag{9}$$

$$^{R}\overset{\leftrightarrow}{N}rbI_{\alpha}^{\rightarrow}(u)=\sum_{i=0}^{n}R_{i,p}(\bar{u}_k)^{R}\overset{\leftrightarrow}{P}_{i}^{\rightarrow}, \tag{10}$$

Each set of data points, Q is the input and P is the output. $^{L}\overset{\leftrightarrow}{P}_{i}^{\leftarrow}$ represents the lower bound of left footprint and $^{L}\overset{\leftrightarrow}{P}_{i}^{\rightarrow}$ is the upper bound of left footprint of control points while $^{R}\overset{\leftrightarrow}{P}_{i}^{\leftarrow}$ is lower bound of right footprint and $^{R}\overset{\leftrightarrow}{P}_{i}^{\rightarrow}$ is the upper bound of right footprint. P_i is the control points generate by crisp data points and $^{T2}\overset{\leftrightarrow}{Q}_i$, $^{T2}\overset{\leftrightarrow}{Q}_i=\{\langle^{L}\overset{\leftrightarrow}{Q}_{i}^{\leftarrow},^{L}\overset{\leftrightarrow}{Q}_{i}^{\rightarrow}\rangle,Q_i,\langle^{R}\overset{\leftrightarrow}{Q}_{i}^{\leftarrow},^{R}\overset{\leftrightarrow}{Q}_{i}^{\rightarrow}\rangle\}$ generated by type-2 fuzzy data points. $^{T2}\overset{\leftrightarrow}{Q}_i$ preserve the same properties as $^{T2}\overset{\leftrightarrow}{P}_i$ and defined in Definition 3.4. $R_{i,p}(\bar{u}_k)$ is a rational B-spline basis function and each of the NURBS curve is generated by using Algorithm 1. For instance, consider a set of fuzzified type-2 fuzzy data points as shown in Table 1, with crisp knot $u=\{0,0,0,0,0.35,0.7,1,1,1,1\}$ and crisp weight $w_i=\{1,1,1,1,1.5,1\}$. This problem is visualized in Figure 2.

Table 1: Data points of type-2 fuzzy NURBS curve.

	${}^{L}\overleftrightarrow{Q}_i^{\leftarrow}$	${}^{L}\overleftrightarrow{Q}_i^{\rightarrow}$	Q_i	${}^{R}\overleftrightarrow{Q}_i^{\leftarrow}$	${}^{R}\overleftrightarrow{Q}_i^{\rightarrow}$
q_0	(2.2,0)	(2.5,0)	(3,0)	(3.5,0)	(3.8,0)
q_1	(0,5)	(0,4.5)	(0,4)	(0,3.6)	(0,3.2)
q_2	(3,5.5)	(3.5,5.7)	(4,6)	(4.2,6.2)	(5,6.5)
q_3	(6.9,4)	(7.3,4)	(8,4)	(8.3,4)	(8.6,4)
q_4	(9.3,6)	(9.5,6)	(10,6)	(10.2,6)	(10.5,6)
q_5	(7,9.1)	(7,9.3)	(7,10)	(7,10.5)	(7,10.9)

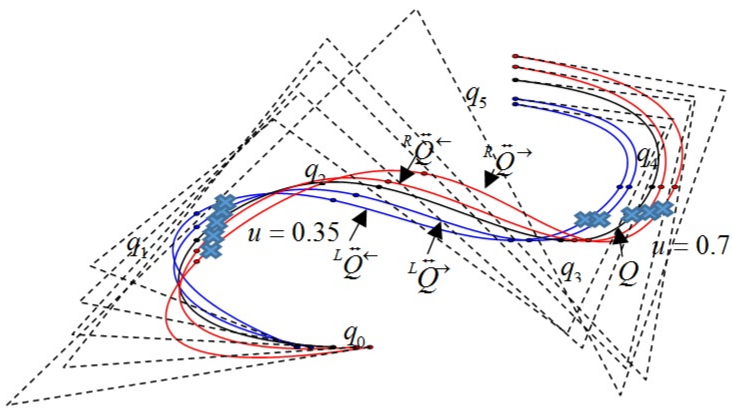

Figure 2: Type-2 fuzzy NURBS interpolation with type-2 fuzzy data points.

Table 2: Data points of type-2 fuzzy NURBS curve after $\alpha-$cut at 0.5.

	${}^{L}\overleftrightarrow{Q}^{0.5\leftarrow}$	${}^{L}\overleftrightarrow{Q}^{0.5\rightarrow}$	Q_i	${}^{R}\overleftrightarrow{Q}^{0.5\leftarrow}$	${}^{R}\overleftrightarrow{Q}^{0.5\rightarrow}$
q_0	(2.6,0)	(2.75,0)	(3,0)	(3.25,0)	(3.4,0)
q_1	(0,4.5)	(0,4.25)	(0,4)	(0,3.8)	(0,3.6)
q_2	(3.5,5.75)	(3.75,5.85)	(4,6)	(4.1,6.1)	(4.5,6.25)
q_3	(7.45,4)	(7.65,4)	(8,4)	(8.15,4)	(8.3,4)
q_4	(9.65,6)	(9.75,6)	(10,6)	(10.1,6)	(10.25,6)
q_5	(7,9.55)	(7,9.65)	(7,10)	(7,10.25)	(7,10.45)

By applying $\alpha-$cut at 0.5, type-2 fuzzy data points is obtain as in Table 2 and the NURBS curve can be visualized as in Fig. 3.

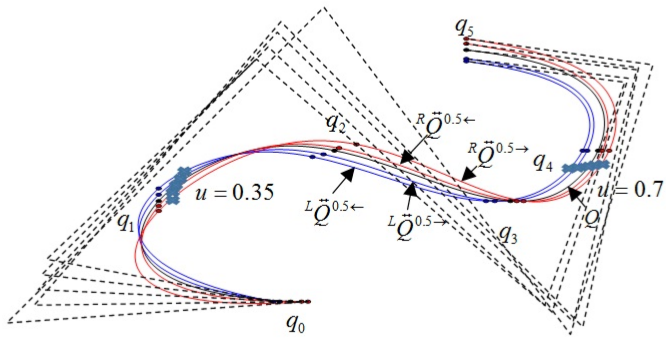

Figure 3: Type-2 fuzzy NURBS interpolation after $\alpha-$cut at 0.5.

After applying centroid type-reduction, type-1 fuzzy NURBS curve data points is obtained as in Table 3 and the curve is visualized in Fig. 4.

Table 3: Data points of type-2 fuzzy NURBS curve after type-reduction.

	${}^{L}\overleftrightarrow{Q}^{0.5}$	Q	${}^{R}\overleftrightarrow{Q}^{0.5}$
q_0	(2.675,0)	(3,0)	(3.325,0)
q_1	(0,4.375)	(0,4)	(0,3.7)
q_2	(3.625,5.8)	(4,6)	(4.3,6.175)
q_3	(7.55,4)	(8,4)	(8.225,4)
q_4	(9.7,6)	(10,6)	(10.175,6)
q_5	(7,9.6)	(7,10)	(7,10.4)

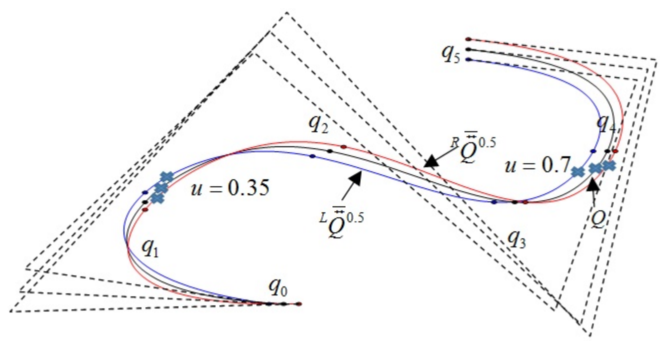

Figure 4: Type-1 fuzzy NURBS curve interpolation after type-reduction.

By applying centroid defuzzification, the crisp data points can be referred in Table 4. The NURBS curve interpolation is visualized in Figure 5.

Table 4: Data points of type-2 fuzzy NURBS curve after defuzzification.

	Q	$\overline{\overline{Q}}^{0.5}$
q_0	(3,0)	(3,0)
q_1	(0,3.817)	(0,4)
q_2	(3.975,5.992)	(4,6)
q_3	(7.925,4)	(8,4)
q_4	(9.958,6)	(10,6)
q_5	(7,10)	(7,10)

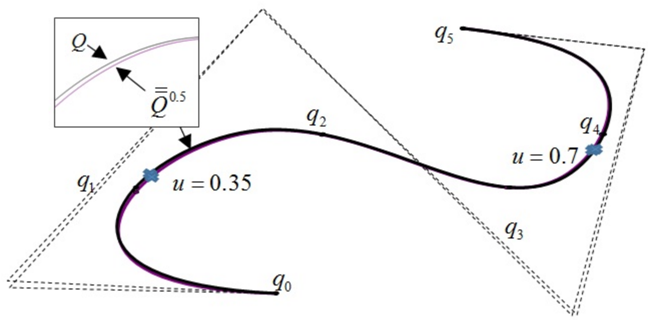

Figure 5: Crisp of type-2 fuzzy NURBS curve interpolation after defuzzification.

Below is the algorithm in order to generate the type-2 fuzzy non-uniform rational B-spline (F2NURBS) curve interpolation model for type-2 fuzzy data points.

Algorithm 1:

1. Step 1. Each upper and lower bound of the left and right footprint of type-2 fuzzy data points, ${}^{L}\overleftrightarrow{Q}^{\leftarrow}$, ${}^{L}\overleftrightarrow{Q}^{\rightarrow}$, Q, ${}^{R}\overleftrightarrow{Q}^{\leftarrow}$, ${}^{R}\overleftrightarrow{Q}^{\rightarrow}$ of NURBS curve is interpolated by using Equation (5).

2. Step 2. $\alpha-$cut is applied on type-2 fuzzy data points and calculated by using Equation (2). Each of the type-2 fuzzy control points of NURBS curve, ${}^{L}\overleftrightarrow{Q}^{\alpha\leftarrow}$, ${}^{L}\overleftrightarrow{Q}^{\alpha\rightarrow}$, Q, ${}^{R}\overleftrightarrow{Q}^{\alpha\leftarrow}$, ${}^{R}\overleftrightarrow{Q}^{\alpha\rightarrow}$ is plotted by using Equation (2).

3. Step 3. Type-reduction is applied and calculated by using Equation (2). Each of the type-1 fuzzy data points, ${}^{L}\overset{\leftrightarrow}{\bar{Q}}{}^{\alpha}$, Q, ${}^{R}\overset{\leftrightarrow}{\bar{Q}}{}^{\alpha}$ is plotted using Equation (5).

4. Step 4. Defuzzification is applied and calculated by using Equation (4). Crisp of type-2 fuzzy NURBS curve, $\bar{\bar{Q}}^{\alpha}$ is then obtained and plotted by using Equation (5).

5. Conclusion

Type-2 fuzzy NURBS model that was discussed in this paper is focused on two dimension curve only. Type-2 fuzzy NURBS curve preserve the same properties as crisp NURBS curve since both left and right footprint of the curve built upon crisp NURBS curve. The type-2 fuzzy NURBS curve control points model is suitable for approximate a curve model. This model could handle type-2 uncertainties or complex uncertainties in control points, knots and weight to obtain desired curve model. In order to solve type-2 uncertainties of data points in NURBS curve interpolation, type-2 fuzzy NURBS curve interpolation method with type-2 fuzzy data points method is used. After $\alpha-$cut and type reduction, type-1 fuzzy data points is obtained and through defuzzification, the result is crisp of type-2 fuzzy data points.

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